A Parallel Integer Programming Approach to Global Routing

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$$\begin{array}{l} \operatorname{GRIP:IP-based global router} \\ \underset{x,s}{\min} \sum_{i=1}^{N} \sum_{t \in \mathcal{T}(T_i)} c_{it} x_{it} + \sum_{\forall e \in E} Q_e o_e & (\operatorname{ILP-GR}) \\ \\ \sum_{\substack{t \in \mathcal{T}(T_i) \\ i=1}}^{N} \sum_{t \in \mathcal{T}(T_i)} x_{it} = 1 & \forall i = 1, \dots, N \\ \\ \sum_{\substack{i=1 \\ i=1}}^{N} \sum_{t \in \mathcal{T}(T_i)} a_{te} x_{it} \leq u_e + o_e & \forall e \in E \\ \\ x_{it} = \{0, 1\} & \forall i = 1, \dots, N, \forall t \in \mathcal{T}(T_i), \\ o_e \geq 0 & \forall e \in E. \end{array}$$

- Solved separately for rectangular subregions by price and bound.
- Limited parallelism since only non-neighboring subproblems can be concurrently solved.



Parallel Router: Overview



Key Steps:

- Sub-problem definition
- •Initial pricing
- Patching
- •Repricing
- Parallel reconnection of neighboring sub-problems

Parallel Router: Initial Steps

- Sub-problem Definition
 - Generate candidate sols using Flute, solve ILP-GR with randomized rounding
 - Bi-partitioning to balance nets
 - Detouring of high overflow nets
- Initial Pricing
 - Inter-region nets connected anywhere on subproblem boundary
 - Larger overflow penalty at boundary.

Parallel Router: Patching



$$\min_{x} \sum_{i=1}^{N} \sum_{t=1}^{|L_i| \times |M_i|} c_{it} x_{it} + \sum_{i=1}^{N} Qs_i \qquad \text{(ILP-PATCH)}$$

$$\begin{cases} \sum_{t=1}^{|L_i| \times |M_i|} x_{it} + s_i = 1 & \forall i = 1, \dots, N \\ \sum_{i=1}^{N} \sum_{t=1}^{|L_i| \times |M_i|} a_{te} x_{it} \le u_e & \forall e \in E'_v \\ x_{it} = \{0, 1\} & \forall i = 1, \dots, N, \forall t = 1, \dots, |L_i| |M_i|. \end{cases}$$

- Q set high to keep slack variable zero
- If si > 0, net i hard to route and given entire window

Parallel Router: Final Steps

- Re-pricing
 - Candidate routes generated with specified window constraints
- Parallel conneting of sub-problems
 - Fix nets inside subproblems and inter-region net "backbones"
 - Connect "backbones" using IP-based price and bound



Frequency Domain Decomposition of Layouts for Double Dipole Decomposition

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Dipole Illumination

- OAI offers better resolution by shifting the diffraction orders
 - Need to capture at least 1st order



Vertical Grating

 Dipole illumination improves resolution for patterns perpendicular to dipole axis



Double Dipole Lithography

Split layout to vertical+horizontal features -> X dipole exposure for vertical -> Y dipole exposure for horizontal

•Incurs cost of 2 masks like DPL

- But, only one etch step, unlike LELE DPL -Intensities from two exposures can interact
- Layout decomposition seems trivial but jogs, line ends hard to classify
- This paper proposes frequency domain decomposition.

DDL Decomposition



$$M(u,v) = F\{m(x,y)\} = \int_{-\infty-\infty}^{\infty} \int_{-\infty-\infty}^{\infty} m(x,y) e^{-j2\pi(ux+vy)} dxdy$$
$$H(u,v) = \begin{cases} 1 & \text{if } \sqrt{u^2 + v^2} < \frac{NA}{\lambda} \\ 0 & \text{otherwise} \end{cases}$$

$$A(x, y) = F^{-1} \{ M(u, v) . H(u, v) \}$$

$$A_X(x, y) = F^{-1} \{ M(u - u_0, v) . H(u, v) \}$$

$$A_Y(x, y) = F^{-1} \{ M(u, v - v_0) . H(u, v) \}$$



$$m_{X}(x,y) = F^{-1} \{ M(u,v) \cdot H_{X}(u,v) \}$$

$$m_{Y}(x,y) = F^{-1} \{ M(u,v) \cdot H_{Y}(u,v) \}$$

Fixing non-binary splits

- For each pixel, magnitude of the two IFTs compared and each pixel is then assigned an exposure accordingly
- If equal, assigned to both exposures
- Aerial image simulation results indicate more robust decomposition with lesser hotspots

