

# Programmable Logic Array

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Liangzhen Lai

# Outline

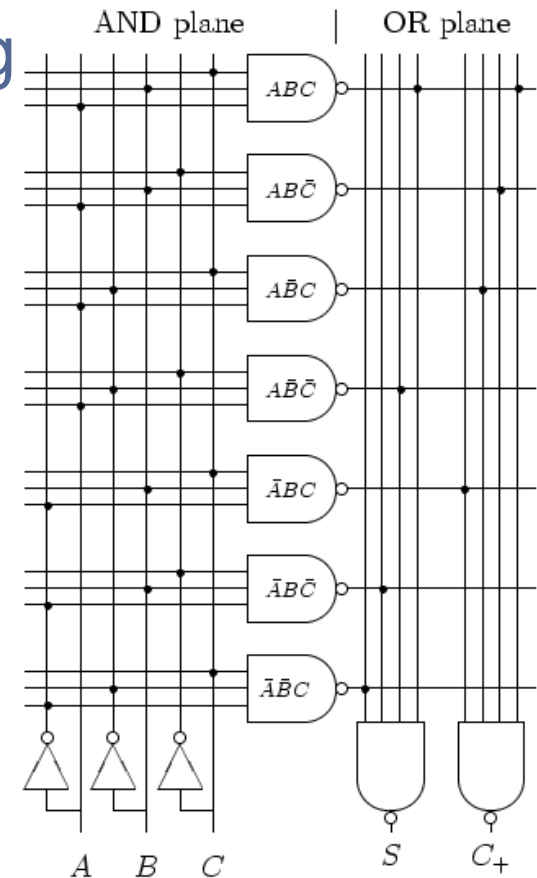
- Introduction
- Anti-fuse
- 2-level logic decomposition
- Multi-valued Applications
- PLA Complexity

# Introduction

- Programmable Logic Arrays (PLAs)
- A set of AND gates linking to OR gates
- Implement Boolean expression using sum-of-product(SOP)
- The figure shows how to implement an adder using PLAs

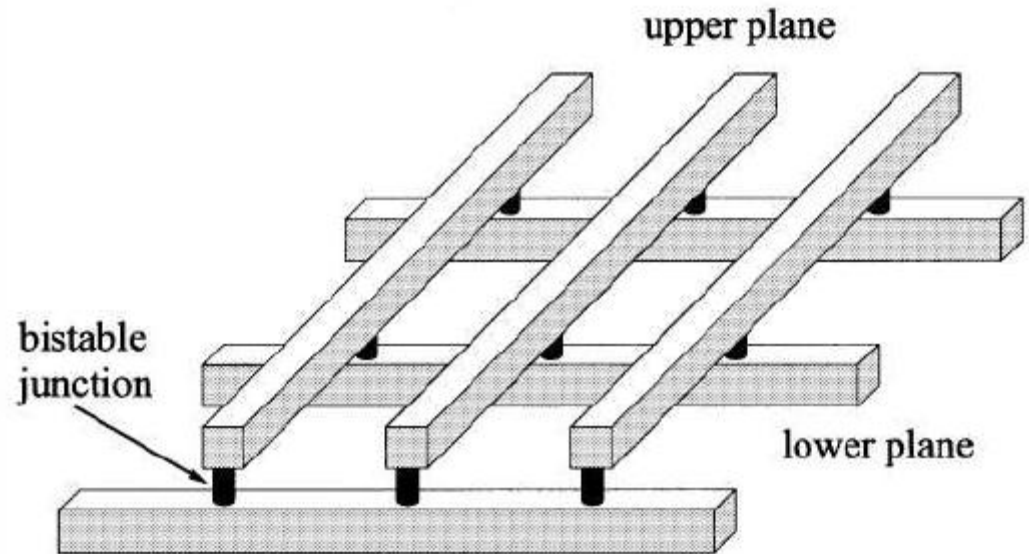
$$S = \overline{\overline{A}}\overline{\overline{B}}\overline{\overline{C}} + \overline{\overline{A}}\overline{\overline{B}}\overline{\overline{C}} + \overline{\overline{A}}\overline{\overline{B}}\overline{\overline{C}} + \overline{\overline{A}}\overline{\overline{B}}\overline{\overline{C}}$$

$$C_+ = \overline{\overline{A}}\overline{\overline{B}}\overline{\overline{C}} + \overline{\overline{A}}\overline{\overline{B}}\overline{\overline{C}} + \overline{\overline{A}}\overline{\overline{B}}\overline{\overline{C}} + \overline{\overline{A}}\overline{\overline{B}}\overline{\overline{C}}$$



# Anti-fuse

- Anti-fuse employs a thin barrier of non-conducting amorphous silicon between two metal conductors.
- Usually in mesh structure
- When a sufficiently high voltage is applied across the amorphous silicon it is turned into a polycrystalline silicon-metal alloy with a low resistance, which is conductive.



## 2-level Logic Decomposition

- Every Boolean logic can be decomposed into product-of-sum (POS) or sum-of-product by Karnaugh map(k-map)

$$\begin{aligned}
 S &= A \oplus B \oplus C = \overline{A}BC + \overline{A}B\overline{C} + A\overline{B}\overline{C} + ABC \\
 &= (A + B + \overline{C})(A + \overline{B} + C)(\overline{A} + B + C)(\overline{A} + \overline{B} + \overline{C})
 \end{aligned}$$

	<b>A'B'</b>	<b>A'B</b>	<b>AB'</b>	<b>AB</b>
<b>C'</b>	0	1	1	0
<b>C</b>	1	0	0	1

# Multiple-valued Application

- The operation of PLAs can be extended to multiple-valued functions
- PLA can perform the following three functions:
  - 1) MIN:  $f(x_1, x_2) = x_1 x_2 (= \text{MIN}(x_1, x_2))$ ,
  - 2) MAX:  $f(x_1, x_2) = x_1 + x_2 (= \text{MAX}(x_1, x_2))$ , and
  - 3) literal:  $f(x_1) = {}^a x_1^b (= r - 1 \text{ if } a \leq x_1 \leq b \text{ and } = 0, \text{ otherwise})$ .

		$x_1$			
		0	1	2	3
$x_2$	0	0	0	0	0
	1	0	1	3	3
	2	0	2	1	0
	3	0	2	1	1

$$f(x_1, x_2) = (1 \text{ }^1 x_1^2 \text{ }^1 x_2^3) + (1 \text{ }^1 x_1^3 \text{ }^3 x_2^3) \\ + (2 \text{ }^1 x_1^1 \text{ }^2 x_2^3) + (3 \text{ }^2 x_1^3 \text{ }^1 x_2^1).$$

# Multiple-valued Application

- The input and output signals has 4 values
- Internal signals have only 2 values
- Special generator/encoder is required

Literal generator.

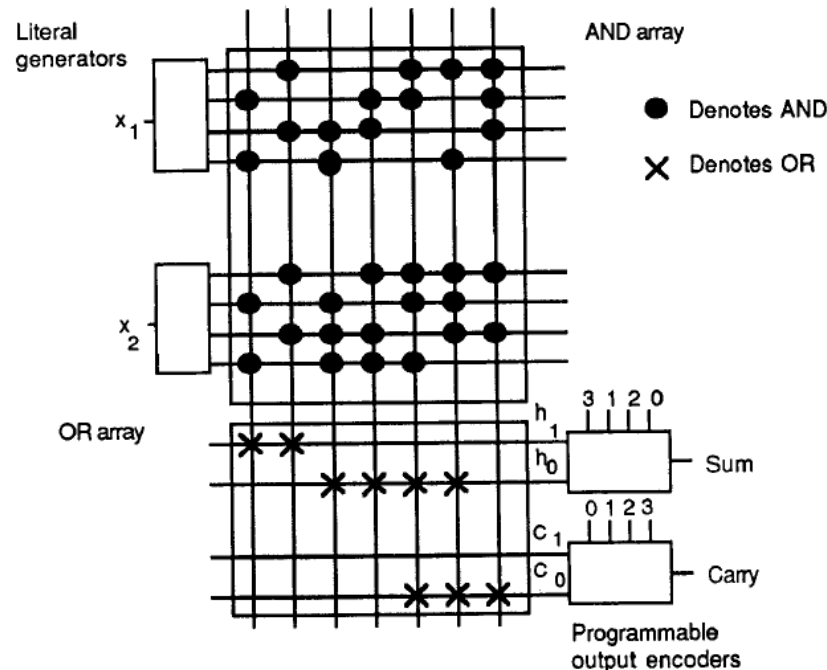
Four-Valued Signal	Two-Valued Signals			
0	0	3	3	3
1	3	0	3	3
2	3	3	0	3
3	3	3	3	0

Output encoding for Sum

Four-Valued Signal	Two-Valued Signals	
0	3	3
1	0	3
2	3	0
3	0	0

Output encoding for Carry.

Four-Valued Signal	Two-Valued Signals	
0	0	0
1	0	3
2	3	0
3	3	3



Four-valued PLA adder.

# PLA Complexity

- Total number of product terms:  $2^n$
- The required product terms can be much less
- The required product terms depends on how many 1's in realized function

..... Upper bound - all prime implicants, Miletto and Putzolu [9].  
 — Upper bound - all prime implicants except certain redundant ones, (14).  
 - - - Upper bound - cover by pairs of 1's plus any needed single 1's, (13).  
 — Lower bound - all essential prime implicants plus certain added implicants, (12).  
 - - - Lower bound - all essential prime implicants, Miletto and Putzulo [9].  
 ..... Lower bound - three types of essential prime implicants, (4).

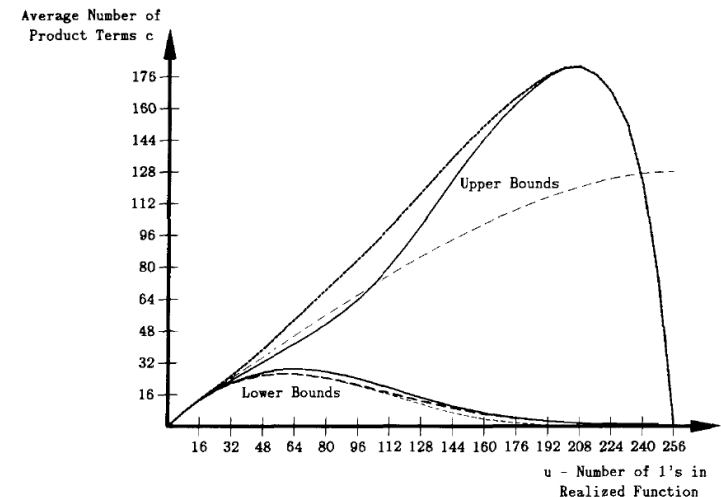


Fig. 2. Upper and lower bounds on the average number of product terms required in the minimal realization of 8-input binary functions versus the number of 1's in the function.



# Reference

[1] E.A. Bender and J. T. Bulter, "On the Size of PLA's Required to Realize Binary and Multiple-valued Functions"

[2] T. Sasao, "Multiple-Valued Logic and Optimization of Programmable Logic Array"