Behavioral Approach for Exact and Approximate Modeling

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I. Markovsky, etc. "Exact and Approximate Modeling of Linear Systems -A Behavioral Approach"

Outline

- Introduction
- Behavioral Approach
- Definitions
- Examples

Introduction

- Modeling: use something simpler, well understood, replicable, applicable to represent/mimic/explain something complex, mysterious, autonomous.
- Key points:
  - Data: by disturbance and observation
  - Constraints: knowledge about the system
  - Evaluation: misfit, latency

Behavioral Approach

- Input-State-Output Approach
  - Classify data into input and output
  - Represent the model by defining the relation between input and output
    - For example: transfer function
- Behavioral Approach
  - Group all data as observation
  - Represent the model using observed data
    - For example: data matrix

Definitions

- U: Universion of all possible outcomes
- D: Observed data
- B: model

A model B is a subset of U which explains D

$$D \subseteq B \subseteq U$$

B_1 is more powerful than B_2 if

$$D \subseteq B_1 \subset B_2 \subseteq U$$

Due to noise or approximation, B may not explain D

Assume D^B is the data predicted by B

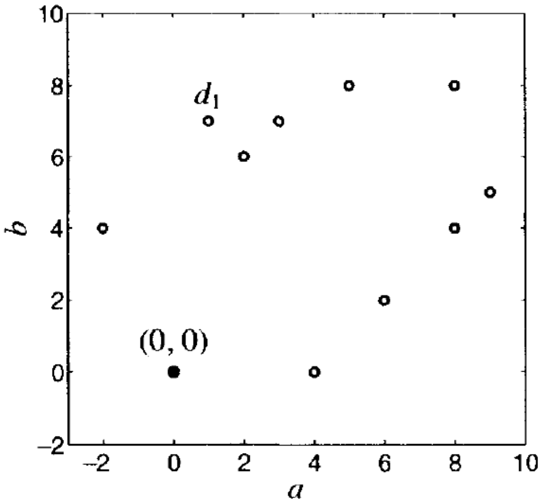
- M: misfit $M(D,B) = \left\| M(d_1, d_1^B), \ldots \right\|$

Example

- We have data D consisting of 2 variables and 10 data points
- We want a linear model that passes

through origin (0,0) which minimize

the misfit M

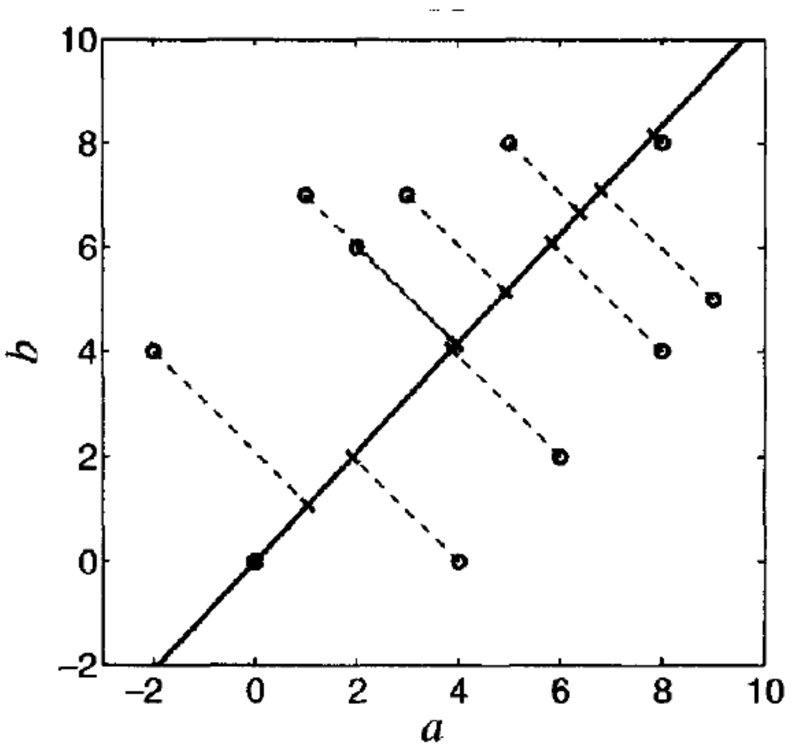


Misfit

- We can define misfit function M(D,B)
- If we define it as 2-norm

$$\min_{B}\{\min_{D^B}\{\left\|D-D^B\right\|_2, D^B \in B\}, (0,0) \in B\}$$

- It is total-least-square (TLS) problem



Solution

Let $D = U \Sigma V^T$ be ranking singular value decomposition

$$U = [U_1 \quad U_2] \qquad \Sigma = \begin{bmatrix} \Sigma_1 & 0 \\ 0 & \Sigma_2 \end{bmatrix} \qquad V = [V_1 \quad V_2]$$

Then we can have $D^B = U_1 \Sigma_1 V_1^T$

Misfit

- We can define misfit function M(D,B)
- If we define it as weighted 2-norm

$$\min_{B}\{\min_{D^B}\{\left\|W_L(D-D^B)W_R\right\|_2, D^B \in B\}, (0,0)\in B\}$$

- It becomes a generalized TLS

Let D_m be the modified data matrix $D_m = \sqrt{w_l}\, D \sqrt{w_r}$

Then the problem is transformed into TLS on D_m