A Parallel Integer Programming Approach to Global Routing

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GRIP: IP-based global router

\[
\min_{x, s} \sum_{i=1}^{N} \sum_{t \in T(T_i)} c_{it} x_{it} + \sum_{e \in E} Q_e o_e \tag{ILP-GR}
\]

\[
\begin{align*}
\sum_{t \in T(T_i)} x_{it} &= 1 \quad \forall i = 1, \ldots, N \\
\sum_{i=1}^{N} \sum_{t \in T(T_i)} a_{te} x_{it} &\leq u_e + o_e \quad \forall e \in E \\
x_{it} &= \{0, 1\} \quad \forall i = 1, \ldots, N, \forall t \in T(T_i), \\
o_e &\geq 0 \quad \forall e \in E.
\end{align*}
\]

- Solved separately for rectangular subregions by price and bound.
- Limited parallelism since only non-neighboring subproblems can be concurrently solved.
Parallel Router: Overview

Key Steps:
• Sub-problem definition
• Initial pricing
• Patching
• Repricing
• Parallel reconnection of neighboring sub-problems
Parallel Router: Initial Steps

• Sub-problem Definition
  - Generate candidate sols using Flute, solve ILP-GR with randomized rounding
  - Bi-partitioning to balance nets
  - Detouring of high overflow nets

• Initial Pricing
  - Inter-region nets connected anywhere on subproblem boundary
  - Larger overflow penalty at boundary.
Parallel Router: Patching

- Q set high to keep slack variable zero
- If $s_i > 0$, net $i$ hard to route and given entire window
Parallel Router: Final Steps

• Re-pricing
  – Candidate routes generated with specified window constraints

• Parallel connecting of sub-problems
  – Fix nets inside subproblems and inter-region net “backbones”
  – Connect “backbones” using IP-based price and bound

![Diagram showing parallel router connections](image-url)
Frequency Domain Decomposition of Layouts for Double Dipole Decomposition

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Dipole Illumination

- OAI offers better resolution by shifting the diffraction orders
  - Need to capture at least 1\textsuperscript{st} order

- Dipole illumination improves resolution for patterns perpendicular to dipole axis
Double Dipole Lithography

Split layout to vertical+horizontal features -> X dipole exposure for vertical -> Y dipole exposure for horizontal

• Incurs cost of 2 masks like DPL
• But, only one etch step, unlike LELE DPL
  - Intensities from two exposures can interact
• Layout decomposition seems trivial but jogs, line ends hard to classify
• This paper proposes frequency domain decomposition.
DDL Decomposition

\[
M(u, v) = F \{m(x, y)\} = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} m(x, y) e^{-j2\pi(ux+vy)} \, dx \, dy
\]

\[
H(u, v) = \begin{cases} 
1 & \text{if } \sqrt{u^2 + v^2} < \frac{NA}{\lambda} \\
0 & \text{otherwise}
\end{cases}
\]

\[
A(x, y) = F^{-1}\{M(u,v)H(u,v)\}
\]

\[
A_X(x, y) = F^{-1}\{M(u-u_0,v)H(u,v)\}
\]

\[
A_Y(x, y) = F^{-1}\{M(u,v-v_0)H(u,v)\}
\]
Fixing non-binary splits

• For each pixel, magnitude of the two IFTs compared and each pixel is then assigned an exposure accordingly

• If equal, assigned to both exposures

• Aerial image simulation results indicate more robust decomposition with lesser hotspots